

Maximum Reachability Orientation of Mixed Graphs

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Abstract

We aim to find orientations of mixed graphs optimizing the total reachability, a problem that has applications in causality and biology. For a given digraph D , we use $P(D)$ for the set of ordered pairs of distinct vertices in $V(D)$ and we define $\kappa_D : P(D) \rightarrow \{0, 1\}$ by $\kappa_D(u, v) = 1$ if v is reachable from u in D , and $\kappa_D(u, v) = 0$, otherwise. We use $R(D) = \sum_{(u,v) \in P(D)} \kappa_D(u, v)$.

Now, given a mixed graph G , we aim to find an orientation \vec{G} of G that maximizes $R(\vec{G})$. We show that this problem is NP-hard and APX-hard, answering a question Hakimi, Schmeichel and Young.

The problem being polynomial time solvable in undirected graphs, we consider the parameterized complexity of the problem with respect to the number k of preoriented arcs of G .

We show that the problem can be solved in time $n^{O(k)}$ and that a $(1 - \epsilon)$ -approximation can be computed in time $f(k, \epsilon)n^{O(1)}$ for any $\epsilon > 0$.